# AS Further Mathematics Unit 3: Further Mechanics A

## Solutions and Mark Scheme

| Question Number | Solution | Mark | AO | Notes |
|---|---|---|---|---|
| 1. (a) | Conservation of momentum | M1 | AO3 | Dimensionally correct |
| | $12 \times 600 = 1600 \times v$ | A1 | AO2 | |
| | $v = \frac{9}{2}$ (ms$^{-1}$) | A1 | AO1 | allow -ve |
| (b) | Energy considerations | M1 | AO3 | |
| | $E = 0.5 \times 12 \times 600^2 + 0.5 \times 1600 \times 4.5^2$ | A1 | AO2 | both expressions correct, Ft $v$ in (a) |
| | $E = 2160000 + 16200$ | | | |
| | $E =$ <u>2176200 (J)</u> | A1 | AO1 | cao |
| | Energy dissipated by eg sound of cannon firing ignored. | B1 | AO3 | oe |
| | In actual fact, quite a lot of energy would be dissipated as sound or heat or in overcoming the friction in the barrel of the cannon. | E1 | AO3 | |
| (c) | Work-energy principle | M1 | AO3 | Used |
| | $F \times d = E$ | | | |
| | $F \times 1.2 = 16200$ | | | |
| | $F =$ <u>13500 (N)</u> | A1 | AO2 | cao |
| | We have not taken into account friction or other resistance to motion which would stop the recoiling part anyway even if no external force is applied. | E1 | AO3 | |
| | So the calculated force is an over estimate of the force required. | B1 | AO3 | |
| | | **[12]** | | |

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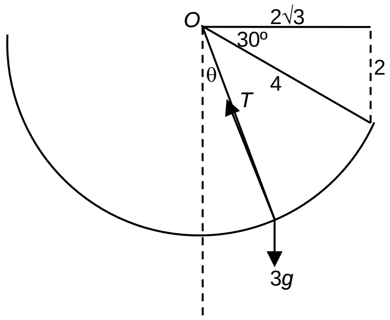
|---|---|---|---|---|
| 2. |  | | | |
| (a) | For *P* falling freely<br>$v^2 = u^2 + 2as,\ u = 0,\ s = 2,\ a = g$<br>$v^2 = 2g\times2$<br>$v = 2\sqrt{g}$ | M1<br><br>A1 | AO3<br><br>AO1 | |
| | When string tightens, *P* has vertical speed $2\sqrt{g}$. The component of this along the string is destroyed and *P* begins to move in a vertical circle with initial speed $2\sqrt{g}\cos30°$<br>$= 2\sqrt{g}\times\frac{\sqrt{3}}{2} = \sqrt{3g}$ | m1<br>A1 | AO3<br>AO1 | |
| (b) (i) | Conservation of energy | M1 | AO3 | KE and PE in dim correct equation |
| | $\frac{1}{2}mv^2 = \frac{1}{2}m\times3g + mg\times4\ (\cos45° - \sin30°)$ | A1<br>A1 | AO1<br>AO1 | KE<br>PE |
| | $v^2 = 3g - 4g + 8g\cos45°$<br>$v^2 = 45.63717(165)$<br>$v = 6.76\ \text{ms}^{-1}\ \ (6.7555289\ldots)$ | A1 | AO1 | |
| (ii) | N2L towards centre | M1 | AO3 | dim correct equation |
| | $T - mg\cos45° = \frac{mv^2}{4}$ | A1 | AO2 | |
| | $T = 3g\left(\frac{1}{\sqrt{2}}\right) + \frac{3}{4}(45.63717)$ | m1 | AO1 | substitute for $v^2$ |
| | $T = 55.02\ \ (55.0168\ldots.)$ | A1 | AO1 | |
| | | **[12]** | | |

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| 3. | $\mathbf{r} = \mathbf{p} + t\mathbf{v}$ | M1 | AO3 | Used |
| | $\mathbf{r}_A = (1 + 2t)\mathbf{i} + 5t\mathbf{j} - 4t\mathbf{k}$<br>$\mathbf{r}_B = (3 + t)\mathbf{i} + 3t\mathbf{j} - 5t\mathbf{k}$ | A1 | AO2 | either correct, any form |
| | $\mathbf{r}_B - \mathbf{r}_A = (2 - t)\mathbf{i} - 2t\mathbf{j} - t\mathbf{k}$ | M1 | AO3 | |
| | $AB^2 = x^2 + y^2 + z^2$ | M1 | AO1 | |
| | $AB^2 = (2 - t)^2 + 4t^2 + t^2$ | A1 | AO1 | |
| | $(AB^2 = 6t^2 - 4t + 4)$ | | | cao |
| | Differentiate | M1 | AO2 | at least 1 power reduced |
| | $\dfrac{\mathrm{d}AB^2}{\mathrm{d}t} = 2(2 - t)(-1) + 10t \ \ (= 12t - 4)$ | | | |
| | $-4 + 2t + 10t = 0$ | m1 | AO2 | equating to 0 |
| | $t = \dfrac{1}{3}$ | A1 | AO1 | cao |
| | $(\text{least distance})^2 = (2 - \dfrac{1}{3})^2 + 5(\dfrac{1}{3})^2$ | | | |
| | least distance $= \sqrt{\dfrac{10}{3}} =$ <u>1.83 (m)</u> | A1<br>**[9]** | AO1 | cao |

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|---|---|---|---|---|
| 4. (a) | $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ | M1 | AO2 | |
| | $\mathbf{v} = 2e^{2t}\,\mathbf{i} + 2\cos(2t)\,\mathbf{j} - 2\sin(2t)\,\mathbf{k}$ | A1 | AO1 | correct differentiation of any one term |
| | | A1 | AO1 | all correct |
| | $\mathbf{v.r} = 2e^{4t} + 2\cos(2t)\sin(2t) - 2\sin(2t)\cos(2t)$ | M1<br>A1 | AO2<br>AO1 | correct dot product |
| | $\mathbf{v.r} = 2e^{4t}$ which is never 0<br>Hence $\mathbf{v}$ and $\mathbf{r}$ are never perpendicular to each other | E1 | AO2 | |
| (b) | $v^2 = (2e^{2t})^2 + (2\cos(2t))^2 + (-2\sin(2t))^2$ | M1 | AO2 | |
| | $v^2 = 4e^{4t} + 4\cos^2(2t) + 4\sin^2(2t)$<br>$v^2 = 4e^{4t} + 4$ | A1 | AO1 | |
| (c) | KE = $0.5 \times 0.4 \times (4e^{4t} + 4)$<br>KE = $0.8(e^{4t} + 1)$ | B1 | AO1 | |
| (d) | WD = change in KE | M1 | AO1 | |
| | WD = $0.8(e^4 + 1) - 0.8(1 + 1)$<br>WD = $0.8(e^4 - 1) = 42.9$ (J) | A1 | AO1 | |
| (e) | Rate of work = $\frac{d}{dt}$(KE) | M1 | AO2 | |
| | Rate of work = $\frac{d}{dt}(0.8(e^{4t} + 1))$ | | | |
| | Rate of work = $3.2\,e^{4t}$ (W) | A1 | AO1 | |
| | | **[13]** | | |

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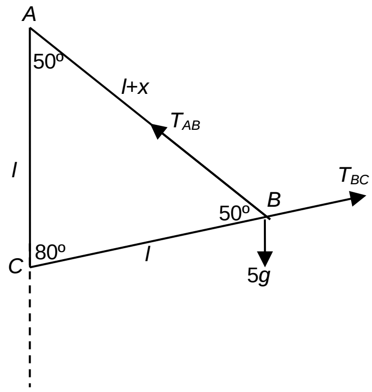
|---|---|---|---|---|
| 5. | Resolve vertically | M1 | AO3 | |
| | $T\cos\theta = mg$ | A1 | AO2 | |
| | N2L towards centre | | | |
| | $T\sin\theta = \dfrac{mv^2}{r}$ | M1 | AO3 | |
| | $T\sin\theta = \dfrac{m\times 4.8^2}{2}$ | A1 | AO2 | |
| | $\tan\theta = \dfrac{4.8^2}{2\times 9.8}$ | m1 | AO1 | |
| | $\theta = 49.61(2371\ldots)^\circ$ | A1<br>**[6]** | AO1 | |
| 6. | <br> | | | |
| (a) | $AB = 2\times 2\cos 50^\circ$ | B1 | AO3 | |
| | Hooke's Law | | | |
| | $T_{AB} = \dfrac{\lambda}{2}(4\cos 50^\circ - 2) = \lambda(2\cos 50^\circ - 1)$ | M1 | AO2 | |
| | $T_{AB} = 0.286\lambda$ (N) | A1 | AO1 | |
| (b) | $EE = \dfrac{1}{2}\dfrac{\lambda\left(4\cos 50^\circ - 2\right)^2}{2}$ | M1 | AO1 | |
| | $EE = 0.0816\lambda$ (J) | A1 | AO1 | |
| (c) | For vertical equilibrium | M1 | AO3 | resolve vertically |
| | $T_{AB}\cos 50^\circ + T_{BC}\cos 80^\circ = mg$ | A1 | AO2 | |
| | $T_{BC}\cos 80^\circ = 49 - \lambda 0.286\cos 50^\circ$ | m1 | AO1 | |
| | $T_{BC} = 282.180 - 1.057\lambda$ (N) | A1 | AO1 | |
| | OR For horizontal equilibrium | (M1) | (AO3) | Resolve horizontally |
| | $T_{AB}\sin 50^\circ = T_{BC}\sin 80^\circ$ | (A1) | (AO2) | |
| | $T_{BC} = \lambda(2\cos 50^\circ - 1)\times\dfrac{\sin 50^\circ}{\sin 80^\circ}$ | (m1) | (AO1) | |
| | $T_{BC} = 0.222\lambda$ (N) | (A1)<br>**[9]** | (AO1) | |

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|---|---|---|---|---|
| 7. | N2L | M1 | AO3 | dim correct, all forces |
| | $T - mg\sin\alpha - R = ma$ | A1 | AO2 | correct equation |
| | $T = \frac{P}{v}$ | B1 | AO3 | used si |
| | $\frac{5P}{16} - 6000 \times 9.8 \times \frac{6}{49} - R = 6000\times2$ | A1 | AO1 | correct equation in $P$ & $R$ |
| | $\frac{5P}{16} - R = 19200$ | | | |
| | N2L with $a$ = 0 | M1 | AO3 | dim correct, all forces |
| | $T - mg\sin\alpha - R = 0$ | A1 | AO2 | correct equation |
| | $\frac{3P}{16} - 6000 \times 9.8 \times \frac{6}{49} - R = 0$ | A1 | AO1 | correct equation in $P$ & $R$ |
| | $\frac{3P}{16} - R = 7200$ | | | |
| | Solving simultaneously | m1 | AO1 | eliminating one variable, Dep. on both M's |
| | $\frac{2P}{16} = 12000$ | | | |
| | $P = 96000; R = 10800$ | A1 **[9]** | AO1 | both answers cao |